MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2017/2018

TMA1201 – DISCRETE STRUCTURES AND PROBABILITY

(All sections / Groups)

13 MARCH 2018
2:30 pm –4:30 pm
(2 Hours)

INSTRUCTION TO STUDENT

1. This question paper consists of 8 printed pages (inclusive of the front page) with 4 questions only. Pages 6 to 8 are appendices for the rules of inference, logical equivalence laws and standard normal distribution table.
2. Attempt **ALL 4** questions. The distribution of the marks for each question is given.
3. Please write all your answers in the answer booklet provided.
4. Show **ALL** of your working steps clearly.
5. **EXCEPT Questions 4b and 4c**, calculators are **ALLOWED** in this examination.

QUESTION 1 (15 MARKS)

a) Let $A = \{x \mid x \text{ is an odd integer} \wedge 0 < x < 10\}$ and
$B = \{x \in Z^+ \mid 12 \text{ is divisible by } x\}$.

i) List the elements in A and B. [1 mark]

ii) Find $A \cap B$, $A - B$ and $P(A \cap B)$. [1.5 marks]

b) Let R be the relation defined on the set $A = \{2,4,5\}$ such that
$R = \{(a,b) \in A \times A \mid (a-b) \text{ divides } 12\}$.

i) List all elements of $A \times A$. [1 mark]

ii) List all elements of R. [1 mark]

iii) Is R antisymmetric? Justify your answer. [1 mark]

iv) Is R a function? Justify your answer. [1 mark]

c) Use **equivalence laws** to show that propositions $\neg q \wedge \neg(p \wedge \neg q)$ and $\neg p \wedge \neg q$ are logically equivalence. State clearly each of the law that you used in your proof. [2.5 marks]

d) Given an argument consisting of the following premises and conclusion:

Premises:

$\exists x \neg P(x)$; $\forall x(P(x) \vee Q(x))$; $\forall x(\neg Q(x) \vee S(x))$; $\forall x(R(x) \rightarrow \neg S(x))$

Conclusion:

$\exists x \neg R(x)$

Use **rules of inference** to show that the argument is true. Please state clearly each of the law that you used in your proof. [6 marks]

Continued .....

QUESTION 2 (10 MARKS)

a) Use mathematical induction to prove that for all integers $n \geq 1$, $4n \geq (2+2n)$. [4.5 marks]

b) Given the recursion

$a_0 = 1$,

$a_1 = 5$,

$a_n = 2a_{n-1} - a_{n-2}$, $n \geq 2$.

Find a_3 and a_4. [1.5 marks]

c) Show that $f(n) = 15n^3 + \frac{1}{2}n$ is $\Theta(n^3)$. [4 marks]

Continued .....

QUESTION 3 (10 MARKS)

a) Let A be the finite-state automaton with states $S = \{\sigma_1, \sigma_2, \sigma_3\}$ and input values $I = \{a, b\}$. The initial state is σ_1 and the accepting state is σ_3. The next-state function is given as follows.

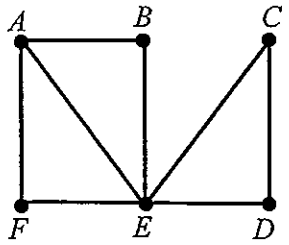
State	Next state	
	a	b
σ_1	σ_1	σ_3
σ_2	σ_2	σ_1
σ_3	σ_3	σ_2

i) Construct the state-transition diagram for the automaton A. [1 mark]

ii) Give a string of four letters that is accepted by A. Give a string of four letters that is not accepted by A. For each case, state clearly the state transitions of the automaton according to the input value starting from the initial state. [2 marks]

iii) Use regular expression to express the language that is accepted by the automaton A. [1 mark]

b) Given the following graph:



Use the depth first search algorithm to find a spanning tree in the above graph. Assume vertex C is the root and the vertices are ordered alphabetically. Show clearly each step of how the algorithm is performed and present your answer in a table with three columns, the first column is the number of steps, the second column is the stack for the search algorithm and the third column is the edges of the resulting spanning tree. [6 marks]

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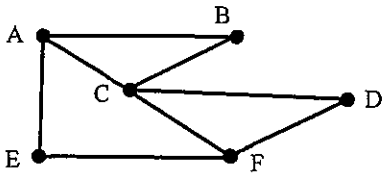
QUESTION 4 (15 MARKS)

a) Show that the graph has an Euler trail and then construct such a trail. [2 marks]



[Important: Calculators are not allowed in Questions 4b and 4c. You must show proper steps on how to simplify and compute those answers.]

b) How many subsets with 6 elements does a set with 10 elements have? [2 marks]

c) In a quiz of discrete mathematics there are 10 multiple choice questions each having answers of A, B and C. Of the answers in these questions, 5 questions have answer A, 3 have answer B and 2 have answer C. If the questions can be positioned in any order, how many different answer keys are possible? [2 marks]

[Note: round the answers in 4d, 4e, and 4f to 4 decimal places.]

d) A phone company is planning to introduce a new smart phone. According to the company marketing report, of new products introduced by the company, 70% have been successes. Furthermore, 80% of their successful products were predicted to be successes, while 25% of failed products were predicted to be successes. Find the probability that this new smart phone will be successful if its success has been predicted. [3 marks]

e) The probability that a patient recover from a rare blood disease is 0.6. If 10 people are known to have contracted this disease, what is the probability that
   i) 5 survive? [1.5 marks]
   ii) at most 8 survive? [1.5 marks]

f) A soft-drink machine is regulated so that it discharges an average of 150 millilitres per cup. If the amount of drink is normally distributed with a standard deviation equal to 20 millilitres,
   i) what fraction of the cups will contain less than 145 millilitres? [1.5 marks]
   ii) how many cups will probably overflow if 175 millilitre cups are used for the next 1000 drinks? [1.5 marks]

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Appendix I

List of Rules of Inference

Addition : $p \Rightarrow (p \vee q)$

Conjunction : $p \wedge q \Rightarrow (p \wedge q)$

Disjunctive Syllogism: $[(p \vee q) \wedge (\neg p)] \Rightarrow q$

Hypothetical Syllogism: $[(p \rightarrow q) \wedge (q \rightarrow r)] \Rightarrow (p \rightarrow r)$

Modus Ponens: $[(p \rightarrow q) \wedge p] \Rightarrow q$

Modus Tollens : $[(p \rightarrow q) \wedge (\neg q)] \Rightarrow \neg p$

Resolution: $[(p \vee q) \wedge (\neg p \vee r)] \Rightarrow (q \vee r)$

Simplification: $[p \wedge q] \Rightarrow p$

List of Logical Equivalence Laws

Associative : (i) $p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r \Leftrightarrow p \vee q \vee r$

(ii) $p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r \Leftrightarrow p \wedge q \wedge r$

Commutative : (i) $p \wedge q \Leftrightarrow q \wedge p$ (ii) $p \vee q \Leftrightarrow q \vee p$

Conversion of Implication: $p \rightarrow q \Leftrightarrow \neg p \vee q$

Distributive : (i) $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

(ii) $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

De Morgan : (i) $\neg(p \wedge q) \Leftrightarrow (\neg p \vee \neg q)$

(ii) $\neg(p \vee q) \Leftrightarrow (\neg p \wedge \neg q)$

Domination : (i) $p \wedge F \Leftrightarrow F$ (ii) $p \vee T \Leftrightarrow T$

Double Negation: $\neg \neg p \Leftrightarrow p$

Idempotent : (i) $p \vee p \Leftrightarrow p$ (ii) $p \wedge p \Leftrightarrow p$

Identity : (i) $p \wedge T \Leftrightarrow p$ (ii) $p \vee F \Leftrightarrow p$

Negation : (i) $p \wedge \neg p \Leftrightarrow F$ (ii) $p \vee \neg p \Leftrightarrow T$

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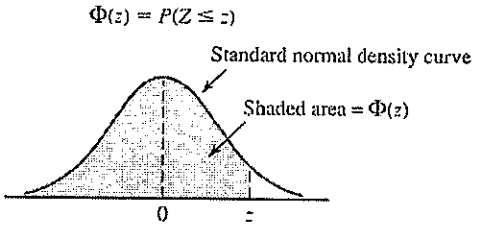
Appendix II Standard Normal Curve Areas

Standard Normal Curve Areas



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0038
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3482
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Continued .....

Standard Normal Curve Areas (Cont.) $\Phi(z) = P(Z \leq z)$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

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